MATHEMATICS MARCH TESTS ASSESSMENT FRAMEWORK

GRADE 12

QUESTION	TOPICS	MARKS
1	***Algebra*** • Factorise • Quadratic formula • Exponents • Inequality • Simultaneous equations • Nature of roots	**20**

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-3)=0$ (2)

1.1.2 $2x^2+1=4x$ (correct to TWO decimal places) (4)

1.1.3 $x^2-2x-3>0$ (4)

1.1.4 $2^{2x}-2^{x+2}-32=0$ (5)

1.1.5 $\sqrt{-2x+4}-x=2$ (4)

1.2 Solve for x and y simultaneously:

$$2x+y=3$$
$$y^2+xy=2$$ (5)

1.3 Consider the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots$

Each factor in the product is of the form $\left(1+\frac{1}{n}\right)$ for $n\geq 2$.

Determine ALL the values of n for which the product will be an integer value. (3)

[27]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\dfrac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4)..(1+a^{512})$

Determine the value of $P \times T$ in terms of a. (3)

[26]

NATURE OF ROOTS

1.3 The roots of the equation $f(x) = 0$ are $x = \dfrac{4 \pm \sqrt{16 - 4m(-m+5)}}{2m}$

Determine the values of m for which the roots will be non-real. (4)

1.4 Show that the maximum value of $\sqrt{-x^2 + 4x + 12}$ is 4. (4)

2	***Number Pattern*** • Quadratic pattern	**09**
3	***Sequences and Series*** • Arithmetic and Geometric • Deriving formulae • Sigma notation • Sum to infinity • Convergence	**28**

QUESTION 2

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; …

2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5)

[14]

QUESTION 2

2.1 Consider the geometric series: $4+2+1+\frac{1}{2}+...$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_∞. (2)

2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k. (5)

[9]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; …

3.1.1 Show that the general term of this number pattern is given by $T_n = \frac{1}{2}n^2 + \frac{5}{2}n$. (3)

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\,527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.
For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41}. (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)
[15]

4	***Functions & Inverse Functions*** • Hyperbola- all forms • Parabola • Use a point to find the equation of the graph • Restriction of the domain • Sketching the graph • Exponential graphs	**25**

QUESTION 6

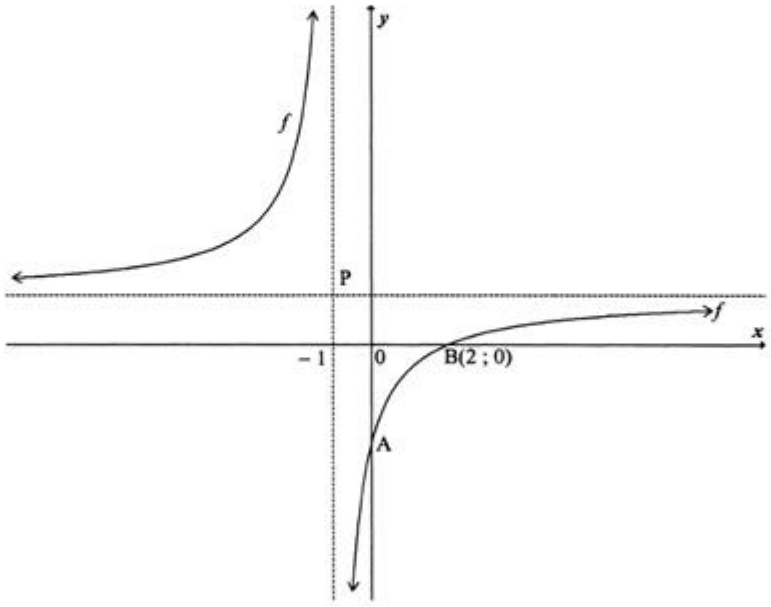
The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbf{R},\ x \neq 5$
- Range: $y \in \mathbf{R},\ y \neq 1$
- Passes through the point $(2\ ;\ 0)$

Determine $f(x)$. [4]

QUESTION 6

A sketch of the hyperbola $f(x)=\frac{x-d}{x-p}$, where d and p are constants, is given below. The dotted lines are the asymptotes. The asymptotes intersect at P and B(2 ; 0) is a point on f.



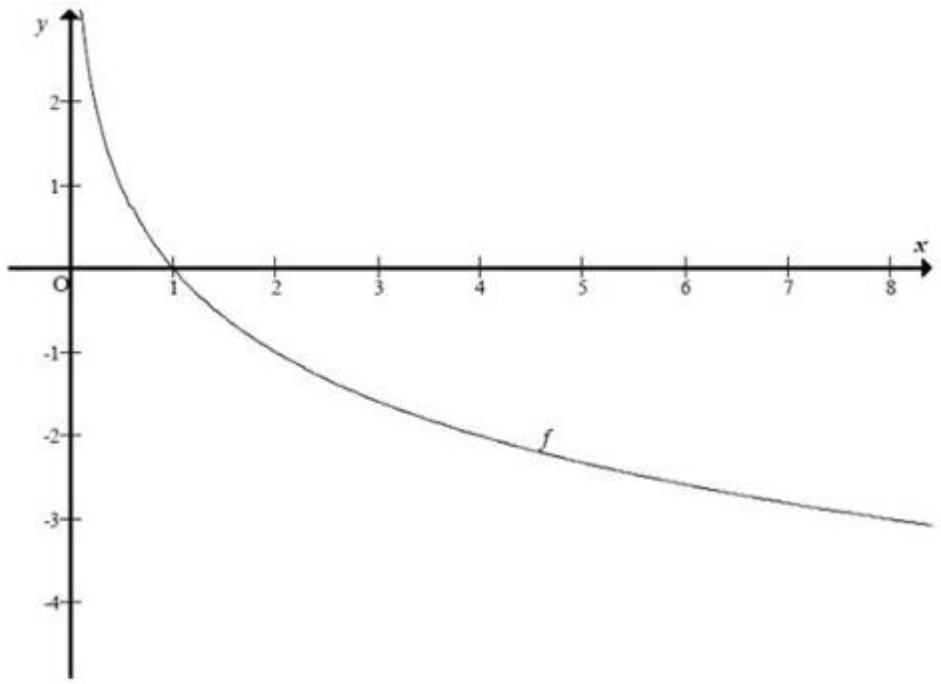
6.1.1 Determine the values of d and p. (2)

6.1.2 Show that the equation of f can be written as $y=\frac{-3}{x+1}+1$. (2)

6.1.3 Write down the coordinates of P. (2)

QUESTION 8

Sketched below is the graph of $f(x) = -\log_2 x$.



8.1 Write down the domain of f. (1)

8.2 Write down the equation of f^{-1} in the form $y = \ldots$ (1)

8.3 Write down the equation of the asymptote of f^{-1}. (1)

8.4 Explain how, using the graph of f, you would sketch the graphs of:

8.4.1 $g(x) = \log_2 x$ (1)

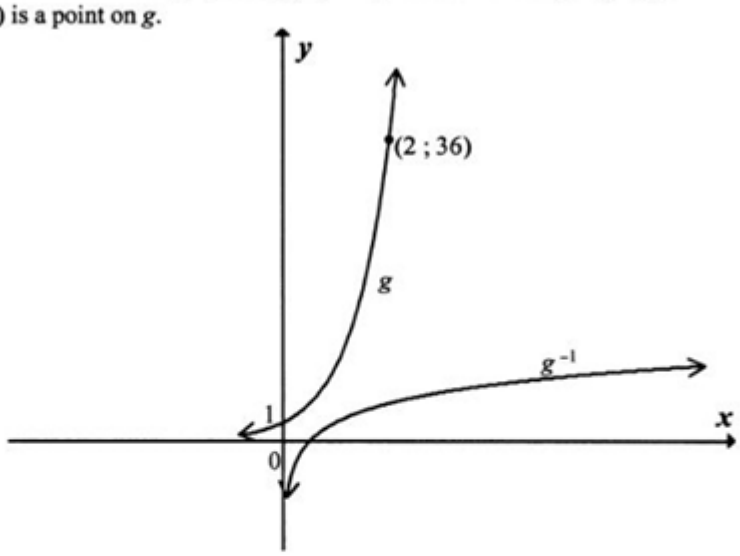
8.4.2 $h(x) = 2^{-x} - 5$ (3)

8.5 Use the graph of f to solve for x where $\log_2 x < 3$. (3)

[10]

QUESTION 5

5.1 Sketched below are the graphs of $g(x)=k^x$, where $k>0$ and $y=g^{-1}(x)$.
(2 ; 36) is a point on g.



5.1.1 Determine the value of k. (2)

5.1.2 Give the equation of g^{-1} in the form $y=...$ (2)

5.1.3 For which value(s) of x is $g^{-1}(x)\leq 0$? (2)

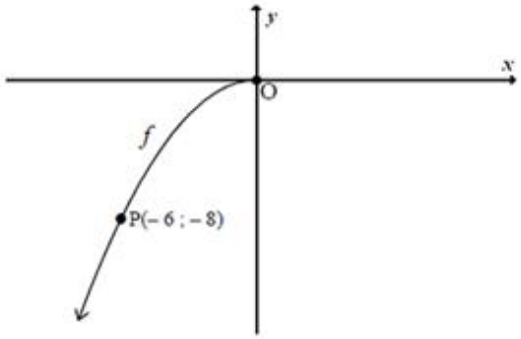
5.1.4 Write down the domain of h if $h(x)=g^{-1}(x-3)$. (1)

5.2.1 Sketch the graph of the inverse of $y=1$. (2)

5.2.2 Is the inverse of $y=1$ a function? Motivate your answer. (2)
[11]

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P(– 6 ; – 8) lies on the graph of f.



6.1 Calculate the value of a. (2)

6.2 Determine the equation of f^{-1}, in the form $y = \ldots$ (3)

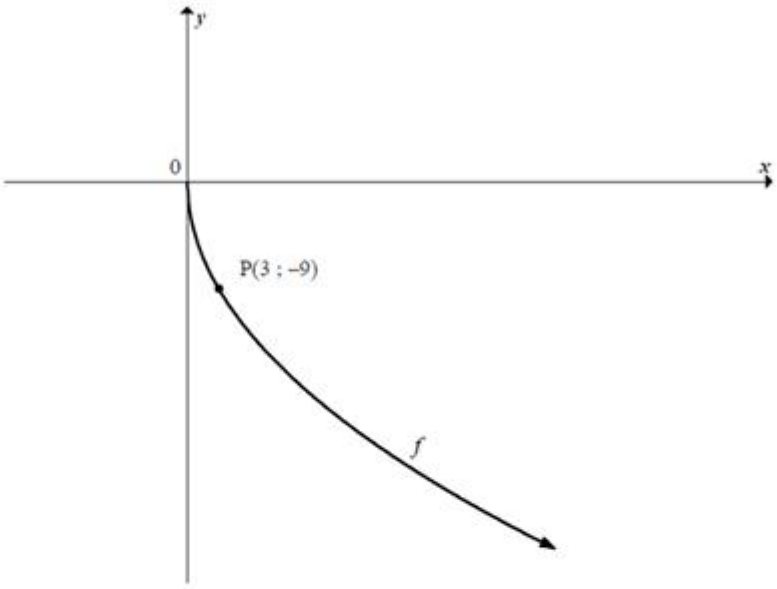
6.3 Write down the range of f^{-1}. (1)

6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from (0 ; 0). (2)

6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axis. Determine the equation of the new function in the form $y = \ldots$ (3)
[11]

QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



5.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)

5.2 Write down the equation of f^{-1} in the form $y = \ldots$ Include ALL restrictions. (3)

5.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

5.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
[9]

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = \ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)
[8]

5	***Trigonometry*** • Identities • Double and compound angles • Reduction formula • Solving triangle – right-angled • Solution of general equations	**18**
	TOTAL	**100**

QUESTION 5

5.1 If $\sin 40° = p$, write EACH of the following in terms of p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Given: $\tan x(1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Prove the above identity. (5)

5.2.2 For which values of x, in the interval $x \in [-180° ; 180°]$, will the identity be undefined? (3)

5.3 Given the expression: $\dfrac{\sin 150° + \cos^2 x - 1}{2}$

5.3.1 **Without using a calculator,** simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

5.3.2 Hence, determine the general solution of $\dfrac{\sin 150° + \cos^2 x - 1}{2} = \dfrac{1}{25}$ (5)

[26]

QUESTION 5

5.1 If $\cos\theta = -\frac{5}{13}$ where $180° < \theta < 360°$, determine, **without using a calculator**, the value of:

5.1.1 $\sin^2\theta$ (3)

5.1.2 $\tan(360° - \theta)$ (2)

5.1.3 $\cos(\theta - 135°)$ (4)

5.2 Simplify the expression to a single trigonometric term: $\frac{2\cos(180° - x)\sin(-x)}{1 - 2\cos^2(90° - x)}$ (6)

5.3 Calculate the value of the following expression **without using a calculator**: $(\tan 92°)(\tan 94°)(\tan 96°) \ldots (\tan 176°)(\tan 178°)$ (4)

[19]

QUESTION 6

6.1 Prove that $2\cos^2(45° + x) = 1 - \sin 2x$. (4)

6.2 Consider the expression: $\sin(A - B) - \sin(A + B)$

6.2.1 Prove that $\sin(A - B) - \sin(A + B) = -2\cos A \sin B$. (2)

6.2.2 Simplify the following expression to a single term: $\sin 4x - \sin 10x$ (2)

6.2.3 Hence, determine the solution for $\sin 4x - \sin 10x = \sin 3x$ for $x \in [0°; 30°]$. (5)

[13]

QUESTION 3

3.1 Given : $\sin\beta=\frac{1}{3}$ where $\beta \in (90°; 270°)$, determine the following by using a sketch and without the use of a calculator:

3.1.1 $\tan\beta$ (3)

3.1.2 $\cos 2\beta$ (2)

3.1.3 $\cos(-\beta-450°)$ (2)

3.2 Simplify the following to a single trigonometric ratio:

$$\frac{4\cos(-x).\cos(90°+x)}{\sin(30°-x).\cos x+\cos(30°-x).\sin x}$$ (6)

3.3 If $cos23° = a$, express the following in terms of a:

3.3.1 $tan203°$ (3)

3.3.2 $sin46°$ (3)

3.4 Determine the values of the following, without using a calculator:

3.4.1 $sin105°$ (4)

3.4.2 $cos69°.cos9° + cos81°.cos21°$ (3)

3.5 Prove the following identity: $\frac{\sin 2x-\cos x}{1-\cos 2x-\sin x}=\frac{1}{\tan x}$ (5)

3.6 Calculate the value of x, if $x \in [-180°; 360°]$

$$cos2x = cosx + 2$$ (7)

[38]